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This derivation depends on the following formula given by Jacobi, (Crelle, Vol. 15):

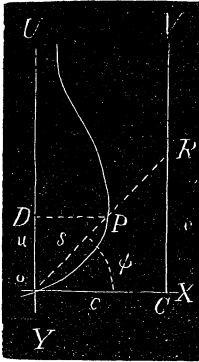
$$\int_0^\pi \phi(\cos t) \cos n t \cdot d t = \frac{1}{1.3.5 \dots (2n-1)} \cdot \int_0^\pi \phi^{(n)}(\cos t) \sin^{2n} t \cdot d t,$$

where  $(n)$  denotes repeated differentiations.

## DISCUSSION OF AN EXPONENTIAL CURVE.

BY IRVING P. CHURCH, B. C. E., NEWBURGH, N. Y.

Construction by Points.—Let  $O C = c$  be the base of any logarithmic system. On  $O U$  and  $C V$ , perpendicular to  $O C$ , lay off  $O D = u$ , and  $C R = v$ , such that  $u = \log v$  in the system whose base is  $c$ . The intersection of  $O R$  and  $D P$ , (parallel to  $O C$ ) fixes  $P$ , a point of the curve.



From the construction we can immediately derive the polar equation of the curve:

$$\begin{aligned} O P = s &= \frac{u}{\sin \phi} = \frac{\log v}{\sin \phi} = \frac{\log . [c \tan . \phi]}{\sin \phi} \\ &= \frac{\log . c + \log \tan \phi}{\sin \phi} \dots\dots\dots(1) \end{aligned}$$

Substituting from  $y = s \sin \phi$ , and  $\tan \phi = \frac{y}{x}$ , we have

for rectangular co-ordinates,

$$y = \log . c + \log \left( \frac{y}{x} \right) = \log . \left\{ \frac{c y}{x} \right\} \text{ or } c^y = \frac{c y}{x},$$

whence we have the final equation of the curve,

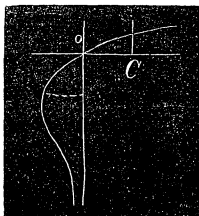
$$x = \frac{y}{c^{y-1}}; \dots\dots\dots(2)$$

and also successively,

$$\frac{dx}{dy} = c^{1-y} (1 - y \log c),$$

$$\text{and } \frac{d^2 x}{dy^2} = c^{1-y} \log c (y \log c - 2); \text{ here } \log c = \text{Nap. log. } c.$$

By reference to the equation (2) we see that if  $c > \text{unity}$  the curve has



the form indicated by our first diagram, which will give a maximum for  $x$ ; also if  $c = \text{unity}$ , (2) becomes  $x = y$ , the equation of a straight line bisecting the first angle, while if  $c < \text{unity}$ , the curve passes to the other side of the line just mentioned, and is tangent to  $Y$  at  $y = -\infty$ , thus, and gives a minimum for  $x$ .

It is now our object to find, when  $c > 1$ , its particular value, such that  $c = x \text{ max.}$  From

$$\frac{dx}{dy} = 0 \text{ we have } y = \frac{1}{l c}, \text{ whence } x \text{ max.} = \frac{1}{c(\frac{1}{l c} - 1)},$$

and putting this equal to  $c$ , we have

$$\frac{1}{l c} = c(\frac{1}{l c}), \dots\dots\dots (3)$$

or  $M = (\text{base})^M$ . (3), solved by successive approximations I find to give, to six decimals

$$c = 1.444667\dots = x \text{ max.},$$

for which value

$$y = \frac{1}{l c} = 2.71828\dots = e = \text{Nap. base.}$$

Now by reference to the first figure and construction of this article, we easily perceive that, if  $C V$  cut the curve, then for any such point of intersection we must have  $u = v$  or  $v = \log v$  in the system whose basis is  $c$ .

Moreover, when  $c = x \text{ max.}$ ,  $C V$  is tangent to the curve, giving

$$u = v = y = e = c^e = 1.444667^e\dots$$

Also noticing that if  $c > 1.444667\dots$   $C V$  cannot intersect the curve; that for

$$c > 1.444667\dots$$

$C V$  intersects it in two points; and that for

$$c = 1$$

$C V$  intersects it but once, we are prepared to make the following deductions:

I. *In logarithmic systems with bases greater than  $e^{\frac{1}{e}} = 1.444667\dots$  there can be no logarithm equal to its antilogarithm or natural number.*

II. *In the system whose base is  $e = 1.444667\dots$ ,*

$$\text{if } e = \frac{1}{l c} \text{ then } l c = \frac{1}{e}; \text{ i. e. } c = e^{\frac{1}{e}},$$

*there is but one such logarithm and its value is  $2.718\dots = e = \text{Nap. base}$ , and it is at the same time the modulus ( $M$ ) of the system,*

$$\text{for } e = \frac{1}{l (1.444667\dots)}.$$

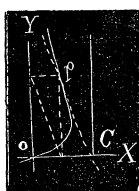
III. *In systems whose bases are less than unity, there is but one such logarithm and it is less than unity.*

Point of Inflection.—If we put

$$\frac{d^2 x}{dy^2} = 0 \text{ we have } y = \frac{2}{c},$$

which substituted in

$$\frac{dx}{dy} \text{ gives } \frac{dx}{dy} = \tan \omega = -c^{1-\frac{2}{c}};$$



also in the value of  $x = \frac{y}{c^{y-c}}$  and we have  $-\frac{x}{y} = -c^{1-\frac{2}{c}}.$

$$\therefore \tan \omega = \left(-\frac{x}{y}\right)$$

for point of inflection. That is, *the tangent to the curve at the point of inflection is parallel to a diagonal of the rectangle formed by the co-ordinates of the point of inflection and the co-ordinate axes.*

### SOLUTIONS OF PROBLEMS IN NO. 3.

Solutions have been received as follows: S.J. Child solved 12; Theo. L. DeLand solved 11; Prof. A. B. Evans solved 11, 12, 13 and 14; Prof. J. M. Greenwood solved 13 and 14; Henry Gunder solved 11 and 12; William Hoover solved 12; Artemas Martin solved 11, 12, 13 and 14; L. Regan solved 12; Walter Siverly solved 11, 12, 13 and 14; S. W. Salmon solved 11 and 12; Prof. J. Scheffer solved 11 and 12; and E. B. Seitz solved 15.

11. "Borrowed a sum of money at 8 per cent. simple interest and loaned it out again at 5 per cent. compound interest; in what time will I gain the amount borrowed."

SOLUTION BY HENRY GUNDER, GREENVILLE, OHIO.

Let the sum borrowed be \$1, and put  $t$  for the time to *gain* this sum. Then by the conditions we have  $(1.05)^t = 2 + .08 t$ . By trial we find  $t = 30 \text{ years} + x$ , a fraction of a year. To find this fraction we have the following equation:  $(1.05)^{30} + (1.05)^{30} \times .05 x = 4.40 + .08 x$ , from which we find  $x = .5735$  years; hence the time is 30y. 6m. 26d.

12. "Given the base  $AC$  of a triangle and the ratio of  $AB$  to  $BC$  to find the locus of the point  $B$  by Geometry."